L^p-Space

Conjugate Number :- Let p, q be any two (+)ve real number

s.t (i) $p > 1$ (ii) $\frac{1}{p} + \frac{1}{q} = 1$

The q is called conjugate to p

If $p=2$, then $q=2$. So that 2 is self-conjugate number

L^p- space :- By $L^p(a,b)$ we mean a class of function $f(x)$, s.t

(i) $f(x)$ is measurable over (a,b)

(ii) $|f|^p$ is L-integrable over (a,b) for $p>0$

i.e $\int_a^b |f|^p \, dx < \infty$ for $p>0$

We denote the class of such functions by the symbol L^p in case mention of interval is not necessary.

Norm of an element of L^p-Space :- Let $f(x) \in L^p(a,b)$ be arbitrary

The norm of $f(x)$ denoted by $\|f\|_p$, is defined as follows

$$\|f\|_p = \left(\int_a^b |f|^p \, dx \right)^{1/p}$$

If $p=1$, then $\|f\|_1 = \|f\| = \int_a^b |f| \, dx$

where (a,b) is finite or infinite interval.

Completeness of L^p-Space :- An L^p-space is said to be complete if every Cauchy sequence in the space is convergent at some point of the space.

i.e for every Cauchy sequence $\langle f_n \rangle$ in the space, there is an element f in the space s.t $f_n \to f$.

A complete normed linear space is called Banach Space.